A Design Theory for Optimum Broadband Reflection Amplifiers

J. O. SCANLAN, MEMBER, IEEE, AND J. T. LIM, STUDENT MEMBER, IEEE

Summary—**The design procedure for optimum broadband negative-resistance amplifiers is given, by reference to the work of Fano [8], on the broadband matching of arbitrary impedances. Complete sets of curves are given which indicate the limits on the gain-bandwidth performance which can be achieved for a particular negative-resistance device, while also showing the ripple in gain and the resulting phase response obtained. The optimum amplifiers are also compared with others of the same class, and it is found that considerable advantage in terms of ripple and phase response can be gained by using nonoptimum designs in certain cases. The paper also includes explicit formula for the element values of the matching network applicable to both optimum and nonoptimum designs. A design example is given for a tunnel-diode amplifier.**

Introduction

THE REFLECTION type negative-resistance amplifier nowadays displays very desirable characteristics for many applications. The amplifier consists of a negative resistance terminating one port of a circulator, which amplifies and reflects a wave incident upon it. The circulator separates the incident and reflected waves. Several devices such as tunnel diodes, parametric amplifiers, and masers display a negative input resistance over some part of the microwave band, together with associated low-noise properties, which render them attractive in this mode of operation. The parametric amplifier and maser suffer the disadvantage of having inherent narrow bandwidths, but have very low noise factors, while the tunnel diode has an inherently broader bandwidth but poorer noise factor. The insertion of a lossless two-port network between the negative-resistance device and the circulator port which it would otherwise terminate yields the possibility of absorbing the energy storage elements associated with the negative resistance which limit the amplifier bandwidth. The design of this lossless two-port network for use with parametric amplifiers has been considered by Matthaei [1], Kuh and Fukada [2], and by Kyhl, McFarlane and Strandberg [3] (in restricted form) for the maser. The application of this principle to the tunnel diode has also received considerable attention [4]–[6].

The design of the two-port coupling network is accomplished by the use of a low-pass lumped circuit prototype network, which is then transformed to the required impedance level and center frequency. Finally each resonant circuit in the lumped element filter has to be approximated by a microwave resonator.

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The authors are with the Department of Electrical Engineering, University of Leeds, Yorkshire, England.

A recent paper by Getsinger [7] gives a design procedure for reflection type amplifiers of the configuration just discussed, based on tables of element values for the low-pass prototype network. In that paper the matching network was chosen to have a Chebyshev response for the gain-frequency characteristic, and the properties of various networks of this class were tabulated. The work presented here is also based on a Chebyshev reponse for the gain-frequency characteristics, but by reference to the work of Fano [8] the limits on the achievable performance are established, and comprehensive design curves are given which enable a rapid design to be accomplished. An amplifier is considered to be optimum if it has the largest bandwidth for a given number of elements in the coupling network and a given negative-resistance device. Closed formulas are given for the element values of the prototype filter from work by Levy [9].

The Low-Pass Prototype Coupling Network

The type of negative-resistance element considered here consists of a negative resistance, $-R$, shunted by a capacity, C. It has been shown [7] that practical negative-resistance devices approximate this ideal model to a good degree of accuracy over bandwidths of less than 20 per cent. The problem to be solved may be stated by reference to Fig. 1 which shows a circulator of characteristic impedance, R_0, a lossless coupling network, N, and the idealized negative resistance. The power gain of this amplifier is the square of the modulus of the reflection coefficient, ρ, at the terminals 22′, but since N is lossless this is also the modulus of the reflection coefficient at the terminals 11′, so that

$$G = |\rho|^2.$$

The gain is ideally required to have a specified value

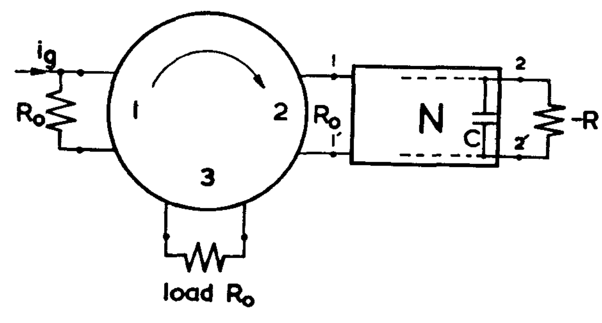

Fig. 1—Circulator and negative resistance with lossless coupling network N.

over some bandwidth, $\omega_1 - \omega_2$, and to be zero outside that band. Transformed to low-pass terms this means a specified gain from $0-\omega_c$, where $\omega_c = \omega_1 - \omega_2$, and zero above ω_c.

Aron [5] has shown that if a lossless network terminated in a positive resistance, R has an input reflection coefficient ρ', and the same network has a reflection coefficient ρ when terminated in $-R$,

$$|\rho'| = \frac{1}{|\rho|}.$$

Bode [10] and Fano [8], have solved the problem of matching an RC load to a pure resistance in the case where R is positive and since the reflection coefficient for the case where R is negative is simply related to the reflection coefficient when R is positive, the results for that case may conveniently be adapted for use with negative resistance.

Bode [10] has shown that (when R is positive)

$$\int_0^\infty \log_e \frac{1}{|\rho'|} d\omega \leq \frac{\pi}{RC} \tag{1}$$

so, in matching problems associated with passive networks where a "worst match" is specified *i.e.*, the maximum value of $|\rho'|$, ($|\rho' \max|$) is specified, the integral in (1) indicates that the minimum value of $|\rho'|$ should not go to zero or the integrand will become infinite at these points thus restricting the bandwidth over which $|\rho' \max|$ is not exceeded. Substituting $|\rho|$ for $1/|\rho'|$ in (1) and since $|\rho| = G^{1/2}$ we get

$$\int_0^\infty \log_e G^{1/2} d\omega \leq \frac{\pi}{RC}. \tag{2}$$

Specifying a maximum value for $|\rho'|$ is equivalent to specifying a minimum gain, G min, so that applying the argument just given for passive networks, (2), shows that for a specified value of G min, the maximum gain should not approach infinity at any point or the effective bandwidth will be reduced.

Returning now to discussion of passive networks, (1) shows that the ideal form of $|\rho'|$ as a function of ω is that shown in Fig. 2. Fig. 3 shows the form of the corresponding ideal transmission characteristic ($|t'|^2 = 1 - |\rho'|^2$). For this ideal characteristic the integral of (1) becomes

$$\omega_c \log_e \frac{1}{|\rho'| \max} = \frac{\pi}{RC} \tag{3}$$

thus yielding the ultimate limit on the bandwidth for a given degree of match and a specified load.

Since the ideal characteristic is obviously not realizable in practice with a finite number of circuit elements, we must seek an approximation which is realizable. The Chebyshev approximation of the type

$$|t'|^2 = \frac{1}{1 + k^2 + h^2 Tn^2(\omega)}$$

is the most suitable, as pointed out by Fano [8], since such an approximation gives the sharpest cutoff (apart from the elliptic function filter) and thereby yields the largest bandwidth in the sense required here.

$Tn(\omega)$ is the Chebyshev function of order n, $1/(1+k^2)$ is the zero-frequency insertion loss and h is a parameter determining the ripple.

Fig. 4 shows $|t'|^2$ as a function of ω. $|\rho'|$ is given by $|\rho'|^2 = 1 - |t'|^2$ *i.e.*,

$$|\rho'|^2 = \frac{k^2 + h^2 Tn^2(\omega)}{1 + k^2 + h^2 Tn^2(\omega)}$$

so that

$$|\rho'|^2 \max = \frac{k^2 + h^2}{1 + k^2 + h^2}$$

and the matching network is constrained by the requirements that the first element is the load capacitance, C.

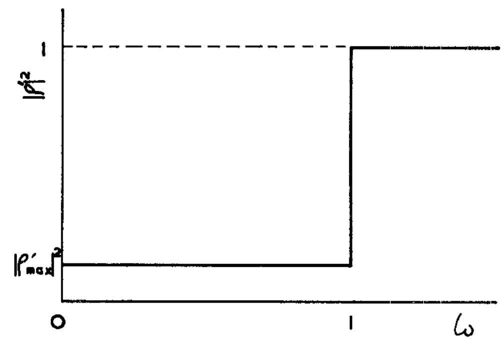

Fig. 2—Ideal reflection coefficient-frequency response.

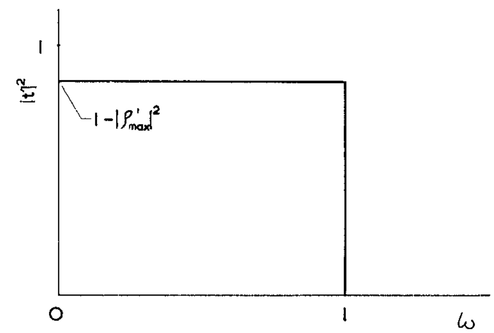

Fig. 3—Ideal transmission-frequency response.

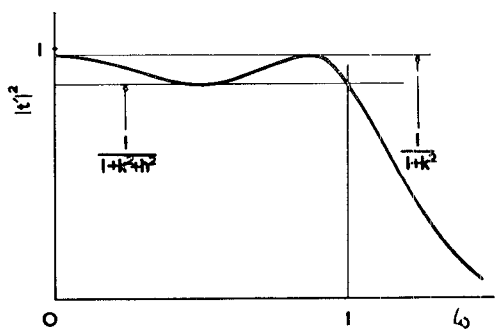

Fig. 4—Chebyshev response with finite insertion loss in the pass band.

Fano [8], then solved the problem of minimizing $|\rho \max|$ subject to this constraint.

Using Fano's notation we define variables a and b as follows

$$\frac{1+k^2}{h^2} = \sinh^2 na$$

$$\frac{k^2}{h^2} = \sinh^2 nb$$

from which

$$|\rho' \max| = \frac{\cosh nb}{\cosh na}.$$

It may be shown [8], that the network constraint imposed by the load capacitance may be expressed in terms of a and b as

$$\frac{2}{\omega_c RC} = \frac{\sinh a - \sinh b}{\sin \frac{\pi}{2n}} \tag{4}$$

minimizing $|\rho' \max|$ subject to this condition yields

$$\frac{\tanh na}{\cosh a} = \frac{\tanh nb}{\cosh b}; \tag{5}$$

a and b are then found by solving (4) and (5) simultaneously.

This work on passive networks is related to the negative-resistance case by the transformation $|\rho'| = 1/|\rho| = 1/G^{1/2}$, so that for the ideal shape of curve previously discussed we have

$$\omega_c \log_e G^{1/2} = \frac{\pi}{RC} \tag{6}$$

which gives the ultimate limit on the gain-bandwidth performance of the amplifier. For the same idealized negative resistance used without a matching network the corresponding expression (for $G \gg 3$ db) is (see Appendix I)

$$(G^{1/2} - 1)\omega_c = \frac{2}{RC}$$

where ω_c is the 3 db bandwidth, thus showing the potential gain-bandwidth advantage obtainable through the use of a matching network.

Again, since the ideal characteristic is not realizable, we use the same approximation as before, but since $|\rho| = 1/|\rho'|$ we have

$$|\rho \min|^2 = G \min = \frac{1+k^2+h^2}{k^2+h^2}.$$

With this transformation Fano's optimization procedure yields an optimum broad-band amplifier in the sense that for a specified G min the bandwidth is the largest possible. Since the optimization procedure yields values for a and b in (4) and (5) the value of G max is obtained, thus there is a particular value of gain ripple corresponding to the largest bandwidth.

Optimum Broad-Band Reflection Amplifiers

Fano's results were presented in the form of graphs showing bandwidth normalized with respect to the RC terminating network and $\log_e (1/|\rho' \max|)$, for various numbers of elements in the matching network, thus enabling k and h to be determined.

Similar curves can be drawn for the negative-resistance terminated network showing minimum gain against normalized bandwidth, the ripple being that corresponding to the optimum bandwidth. The normalization factor on the bandwidth is $1/(2\pi RC)$ the cutoff frequency of the load. Fig. 5 shows the curves for various numbers of elements in the matching network, and it is seen that the results converge rapidly towards the infinite element limit for n greater than 8. These results enable the gain-bandwidth limitation for any particular negative-resistance device and a given number of elements to be determined. Correspondingly, they enable one to say how many elements are required for a given specification of gain and bandwidth.

In the case of the passively terminated network no attention is given to the ripple which results from Fano's optimization procedure since it is usually unimportant.

This ripple is

$$\left|\frac{\rho' \max}{\rho' \min}\right|.$$

However, in the case of the reflection amplifier the corresponding ripple is

$$\delta = \left|\frac{\rho' \min}{\rho' \max}\right|^2 = \frac{G \max}{G \min},$$

and this determines the ratio of the maximum gain to the specified minimum gain in the amplifier pass band.

This ratio is given by

$$\frac{G \max}{G \min} = \frac{(1+k^2)(k^2+h^2)}{k^2(1+k^2+h^2)},$$

and since h and k are determined from Fano's results, the corresponding gain ripple can be found. Figs. 6 and 7 show curves of gain against ripple and ripple against bandwidth for various numbers of elements. The three sets of curves in Figs. 5, 6, and 7 give the complete performance in terms of gain, bandwidth, and ripple for the optimum negative-resistance reflection type amplifier.

In order to complete the design once a particular n has been chosen, the element values of the matching network are required. These can be found from the recurrence relationships derived by Levy [9] from Takahasi's results [11]. Referring to Fig. 8 the recurrence relations are

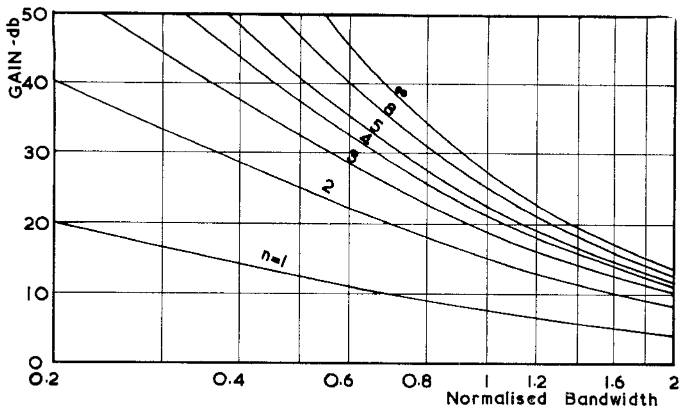

Fig. 5—Gain-bandwidth curves for optimum amplifiers.

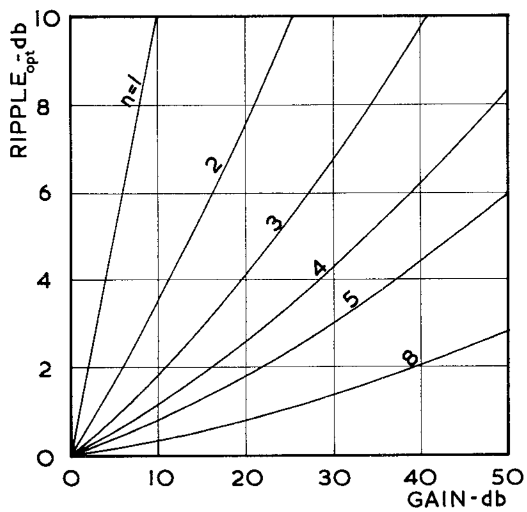

Fig. 6—Gain-ripple curves for optimum amplifiers.

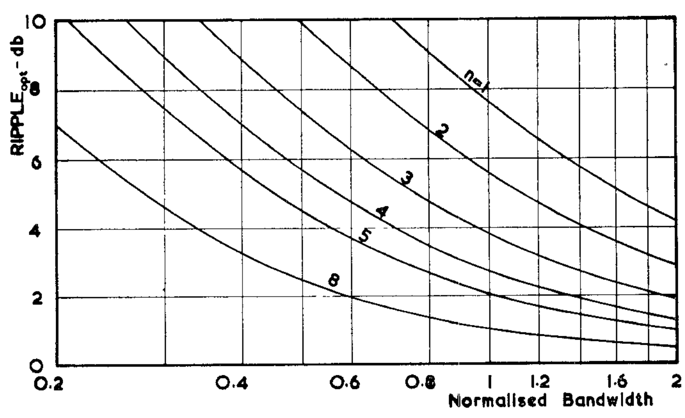

Fig. 7—Bandwidth-ripple curves for optimum amplifier.

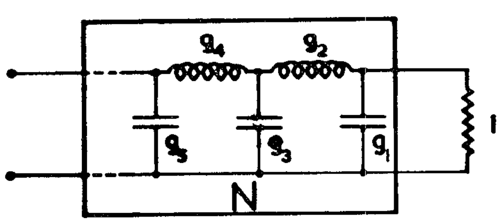

Fig. 8—Low-pass prototype filter.

$$g_1 = \frac{2 \sin \dfrac{\pi}{2n}}{x - y} = RC\omega_c$$

$$g_r g_{r+1} = \frac{4 \sin \dfrac{2r-1}{2n}\pi \cdot \sin \dfrac{2r+1}{2n}\pi}{x^2 + y^2 + \sin^2 \dfrac{r\pi}{n} - 2xy \cos \dfrac{r\pi}{n}}$$

$$r = 1, 2, 3 \cdots n - 1.$$

x and y are given by

$$x = \sinh a = \sinh 1/n \sinh^{-1} \left\{ \frac{\delta(G \min - 1)}{\delta - 1} \right\}^{1/2}$$

$$y = \sinh b = \sinh 1/n \sinh^{-1} \left\{ \frac{G \min - 1}{G \min (\delta - 1)} \right\}^{1/2}$$

where a and b are defined in the previous section and ω_c is the actual bandwidth.

G min and δ are found from Figs. 5 and 7, and the use of these results will be given in a later section.

The Gain Ripple in Reflection Amplifiers

In the previous section, curves of ripple against gain for optimum reflection amplifiers were given. A consideration of these results shows that the ripples obtained in the optimum case are often rather excessive. Gain flatness requirements may prohibit the use of the optimum network and so an investigation of amplifiers having Chebyshev response in the same way as the optimum ones but having nonoptimum ripples must be carried out. This, of course, will reduce the bandwidth, and the question to be answered is whether reducing the ripple level for a specified minimum gain causes much resultant loss of bandwidth. Fig. 9 shows curves of bandwidth against ripple for various fixed gains and various numbers of elements, from which it may be seen that reduction of the ripple level has little effect on the bandwidth achieved. For example, with 20 db minimum gain, and $n=4$, the optimum amplifier has a ripple of 2.6 db, and a normalized bandwidth of 1.07. The ripple can be reduced to 0.58 db for a loss of bandwidth of 5 per cent. Similarly, in other cases, a substantial reduction can be made in the ripple level without incurring much loss of bandwidth. In order to have complete information about the various choices of ripple available, one needs to know the phase characteristics corresponding to the different ripple levels. These can be found in the following manner:

$$|t'(j\omega)|^2 = \frac{1}{1 + k^2 + h^2 Tn^2(\omega)}$$

so that

$$|\rho'(j\omega)|^2 = \frac{k^2 + h^2 Tn^2(\omega)}{1 + k^2 + h^2 Tn^2(\omega)}.$$

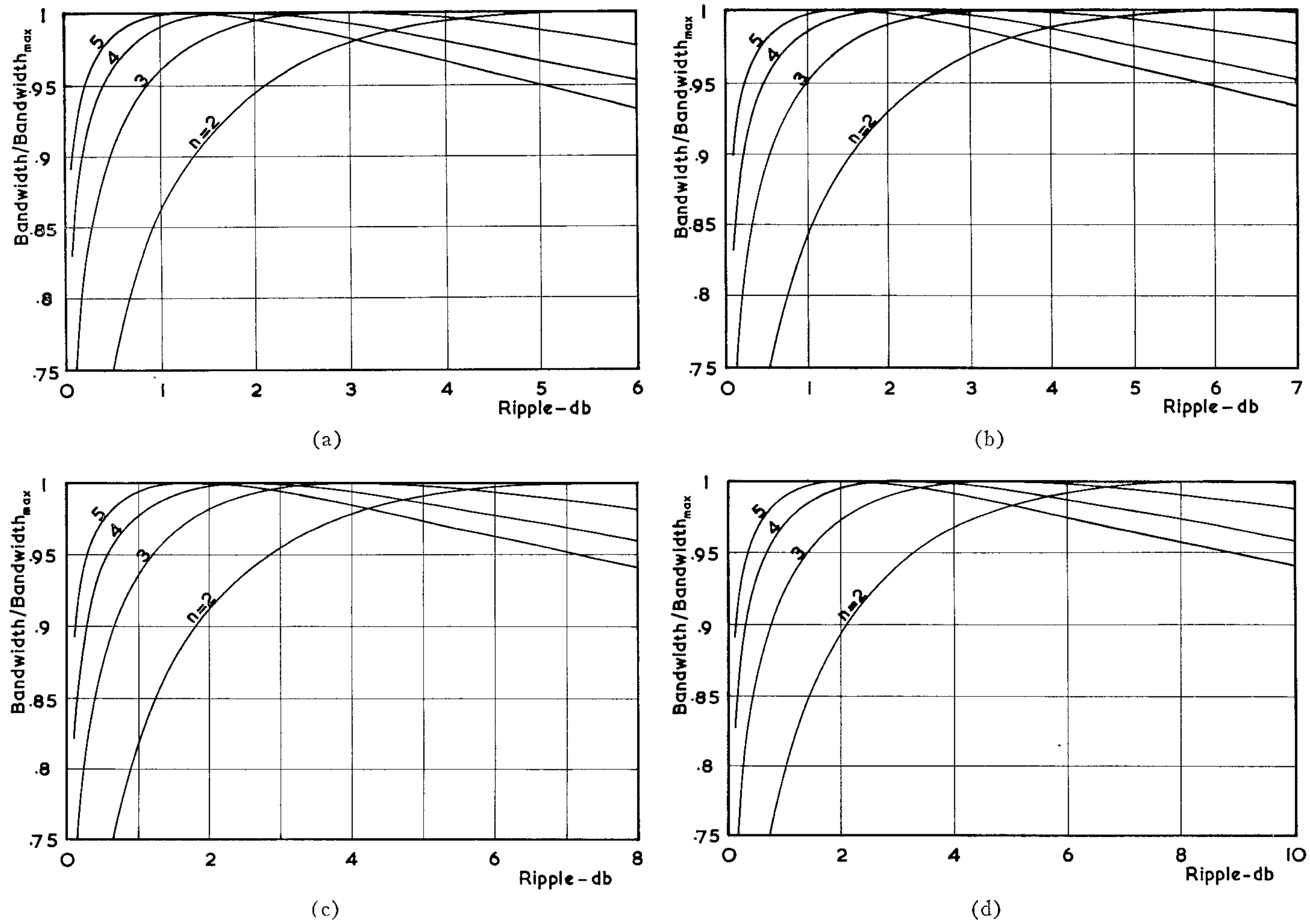

Fig. 9—Bandwidth-ripple curves for various numbers of elements, and minimum gains of (a) 15 db; (b) 17 db; (c) 20 db; (d) 23 db.

$\rho'(s)$ is found from the left-hand plane poles and zeros of $|\rho(j\omega)|^2=\rho'(s)\ \rho'(-s)|_{s=j\omega}$ which are, respectively,

$$-\sin\left\{\frac{(2m-1)\pi}{2n}\right\}\sinh\frac{1}{n}\sinh^{-1}\frac{\sqrt{1+k^2}}{h}+j\cos\left\{\frac{(2m-1)\pi}{2n}\right\}\cosh\frac{1}{n}\sinh^{-1}\frac{\sqrt{1+k^2}}{h},$$

and

$$-\sin\left\{\frac{(2m-1)\pi}{2n}\right\}\sinh\frac{1}{n}\sinh^{-1}\frac{k}{h}+j\cos\left\{\frac{(2m-1)\pi}{2n}\right\}\cosh\frac{1}{n}\sinh^{-1}\frac{k}{h}$$

where $m=1, 2, 3, \cdots n$.

Now if

$$\rho'(s)=\frac{m_1+n_1}{m_2+n_2},$$

where $m_1\ m_2$ are even and $n_1\ n_2$ are odd, it may readily be shown that $\rho(s)=1/\rho'(-s)$, so that the group delay responses of $\rho(s)$ and $\rho'(s)$ are identical. Thus the group delay $tg(=d\phi/d\omega)$ is given by

$$\left[\frac{m_1n_1'-n_1m_1'}{m_1^2+n_1^2}-\frac{m_2n_2'-n_2m_2'}{m_2^2+n_2^2}\right]_{s=j\omega}.$$

Prime indicates derivative with respect to s. Fig. 10 shows group delay-frequency curves for a minimum gain of 20 db, with various numbers of elements and ripples, from which it is seen that a considerable improvement in group delay performance is obtained from use of nonoptimum ripples. The ripples chosen correspond to optimum bandwidth, 95 per cent optimum bandwidth, and 90 per cent optimum bandwidth.

Design Procedure

In this section the design of an optimum broad-band tunnel-diode amplifier will be considered by reference to the curves already given, and some variations with nonoptimum bandwidth will also be discussed.

Let us assume that a simple singly resonant tunnel-diode amplifier has the response curve (b) shown in Fig. 11, when the precautions outlined by Getsinger [7], have been taken to obtain the widest possible bandwidth. It is not implied that the actual performance given by this curve could necessarily be obtained with a

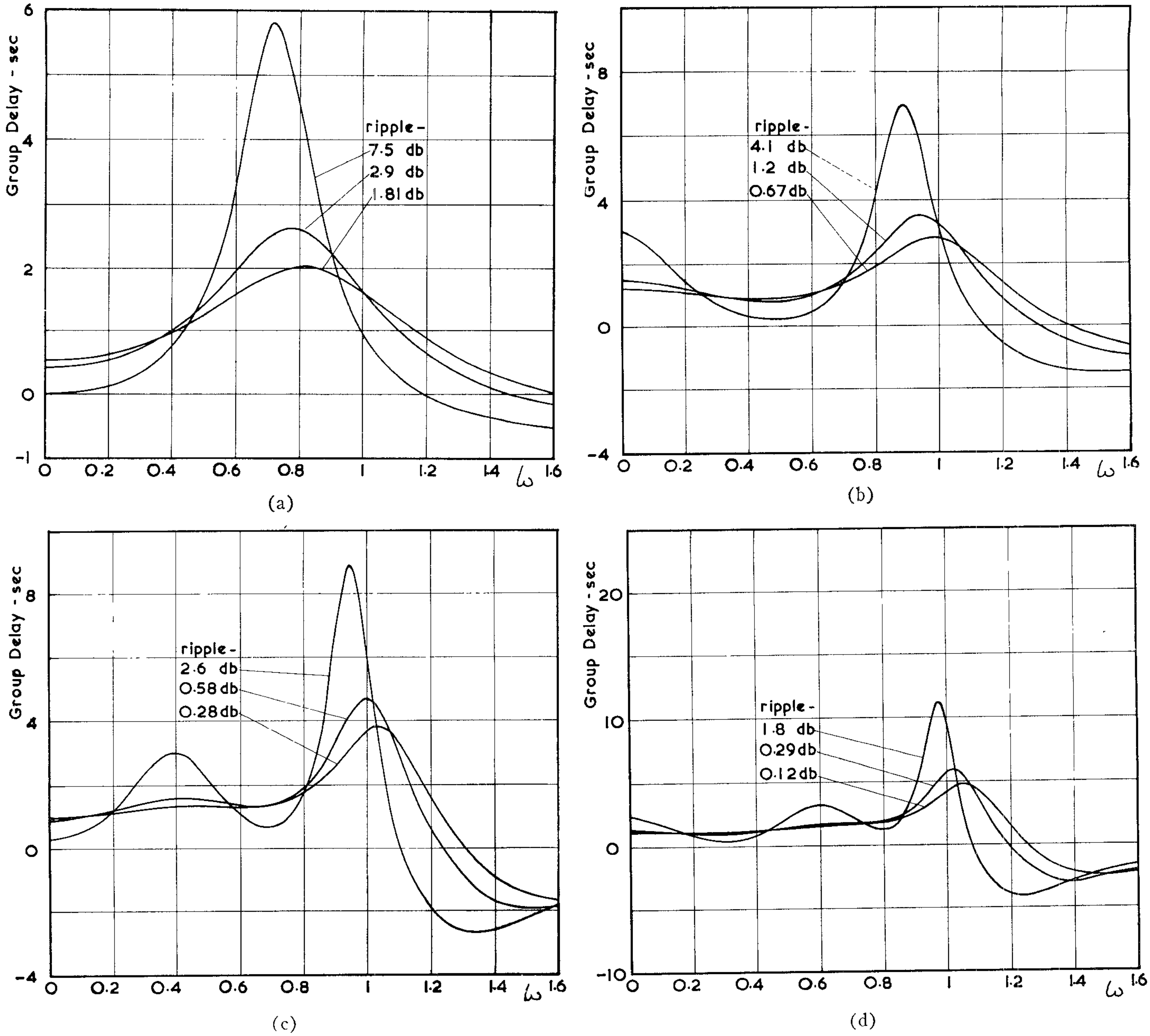

Fig. 10—Group delay response of broad-band amplifiers with 20 db minimum gain and (a) $n=2$; (b) $n=2$; (c) $n=4$; (d) $n=5$.

practical diode. The "unlikely" value of maximum gain is explained in Appendix II. It is assumed that this curve has been obtained by tapering the circulator impedance to a value R_0, which is known. It is desired to approximate the diode by a parallel combination of $-R$ and C, which can be done to a fair degree of accuracy [7]. The quantity of interest is the RC product. This can be obtained from the curve by a procedure which is equivalent to Getsinger's [7] but involves the 3 db bandwidth (B c/s) rather than the half-db-gain bandwidth.

It is shown in Appendix I that if G is the center frequency power gain

$$(G^{1/2} - 1)(1 - 2/G)^{1/2}B = \frac{1}{\pi RC}.$$

From Fig. 11, $G=20.89$ db and $B=1.25$ kmc,

$$RC = 25.4 \times 10^{-12} \text{ sec.}$$

To calculate the potential bandwidth to be obtained at this gain we make use of (6) which yields $B=8.2$ kMc with an infinite number of elements in the matching network. For any finite number of elements we refer to Fig. 5. The normalizing frequency for the bandwidth scale is $1/2\pi RC=6.27$ kMc, so values on the bandwidth scale are fractions of 7.27 kMc in this case. Thus, for example, with 20.89 db gain $n=2$ gives a bandwidth of 4.14 kMc, $n=3$ gives 5.58 kMc and $n=4$ gives 6.65 kMc. Referring to Fig. 6 we find the corresponding ripples to be 7.5 db, 4.1 db, and 2.6 db. Now since the ripples are rather high for most applications we should examine what is the cost in terms of loss of bandwidth of reducing these ripple levels. Referring to the curves of Fig. 9(c) for 20

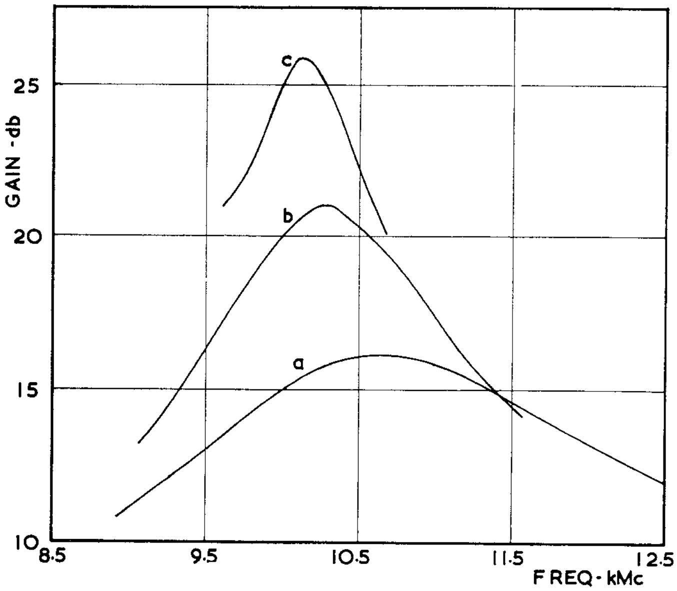

Fig. 11—Gain-frequency response of single tuned tunnel-diode amplifiers.

db gain, which is the nearest to our value of 20.89 db, and assuming that the same reduction in bandwidth will occur so, for example, with $n=3$ if the bandwidth is reduced by 5 per cent compared to the optimum the ripple is reduced from 4.1 db to 1.2 db, with the corresponding improvement in group delay response as shown in Fig. 10(b).

Taking the specified minimum gain of 20.89 db and the single tuned amplifier of Fig. 11, together with a ripple of 1.2 db and a bandwidth corresponding to 95 per cent of the optimum, *i.e.*, 5.3 kMc, the previous discussion shows that $n=3$ will meet the specification, and it remains only to calculate the element values of the coupling network required. The curves given earlier would clearly indicate whether a particular specification was possible at all, and if so, the number of elements required.

In order to calculate the element values, x and y of (7) must first be calculated. These turn out to be

$$x = 1.64; \qquad y = 0.454, \text{ respectively.}$$

As a check on x and y, we can compute $RC\,\omega_c$ which turns out to be 0.844 instead of 0.89, the discrepancy being due to the use of the curves for 20 db instead of 20.89 to calculate bandwidth and ripple. We shall assume x and y to be correct. If the resultant response is not acceptable, one can construct curves for a gain of 20.89 db and find x and y exactly. In calculating the values of the other elements we can take g_1 as 0.89, the correct value, together with the values of x and y just calculated. This then gives

$$g_2 = 0.776$$

$$g_3 = 0.59.$$

Now the circulator admittance g_0 is given by

$$\frac{g_0 + 1}{g_0 - 1} = G^{1/2}\,\text{max} \quad (n \text{ even})$$

$$\frac{g_0 + 1}{g_0 - 1} = G^{1/2}\,\text{min} \quad (n \text{ odd}).$$

In our case $g_0 = 1.2$.

All impedance levels are normalized to 1 Ω negative resistance. If, as previously mentioned, the circulator impedance which gave the original curve is known, this corresponds to an n odd choice. That is to say that if this admittance is g_0', the negative admittance is given by

$$\frac{g_0' + g}{g_0' - g} = G\,\text{min}^{1/2},$$

hence g is found and the values in the filter are unnormalized, with respect to the magnitude of the negative resistance. The remaining steps are to unnormalize with respect to bandwidth, and to resonate each element to the required center frequency.

As a second design example let us assume that the maximum ripple in the gain response is limited to 1 db. We take the minimum gain as 20.89 db and assume that a bandwidth of 3 kMc is required starting with the singly-resonant amplifier with the response curve, line *b* of Fig. 11.

3 kMc expressed as fraction of 6.27 kMc is 0.48, and from Fig. 5 the largest bandwidth obtainable with 20.89 db gain is 0.66 for $n=2$ and 0.89 for $n=3$, with corresponding ripples of 7.85 db and 4.35 db (from Fig. 6). Referring now to Fig. 9(c), which is drawn for a minimum gain of 20 db, the nearest to our value of 20.89 db, we can see that for 1 db ripple the bandwidth is reduced to 0.818 of the maximum for $n=2$ and 0.935 of the maximum for $n=3$. These then yield bandwidths of 3.38 kMc and 5.2 kMc, respectively. Thus our design specification can be realized with $n=2$ and the remaining steps are the same as for the previous example.

Appendix I

Since we are considering the tunnel diode to be represented by a capacitance in parallel with frequency-independent negative resistance, we may simplify Getsinger's [7] slope parameter x to $c\omega_0$ as pointed out by him. Also, instead of taking the half-decibel-gain bandwidth we may take the 3 db bandwidth to specify the response curve.

The power gain on this assumption is given by

$$G = \left| \frac{g_0 + g - jc\omega_0\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)}{g_0 - g + jc\omega_0\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)} \right|^2,$$

and the 3 db bandwidth is given by the difference between the ω values at which G is reduced to half G_0 where

$$G_0 = \left(\frac{g + g_0}{g - g_0}\right)^2.$$

Solving this we get for the 3 db bandwidth in cycles per second

$$B = \frac{1}{\pi RC(G^{1/2} - 1)(1 - 2/G)^{1/2}},$$

and this is the equation used in the text to calculate RC.

Appendix II

In the design of high-frequency tunnel-diode amplifiers, some difficulty arises in determining the gain and center freqeuncy. The expression for gain quoted in Appendix I should read

$$G = \left|\frac{g_0 + g(\omega) - jB(\omega)}{g_0 - g(\omega) + jB(\omega)}\right|^2$$

where $-g(\omega)$ and $B(\omega)$ are the input conductance and susceptance of the tunnel diode, respectively.

The condition for maximum gain is not simply that $B(\omega) = 0$ as can readily be seen by taking the derivative of G, and so the maximum gain is not only a function of $g(\omega_0)$ but of $B(\omega_0)$ as well. To illustrate this effect a particular tunnel-diode equivalent circuit has been assumed, and nominal gains of 15 db, 20 db and 25 db at 10 kMc have been set by choosing appropriate values of g_0 on the assumption that $B(\omega) = 0$ at 10 kMc. The actual response curve in each case was then computed and these curves are shown in Fig. 11. It may be seen that maximum gain in each case did not occur at 10 kMc [where $B(\omega) = 0$] but at some higher frequency, and the actual maximum gain exceeded the nominal value set at 10 kMc. Thus in order to calculate the tuning inductance and circulator impedance for a given center frequency and gain, it is not sufficient to resonate the diode at the required frequency, and then calculate g_0. The exact equation for G must be used, and the error incurred in using the simplified procedure can be seen from Fig. 11. This discrepancy is not large for the equivalent circuit of the diode assumed in computing the curves of Fig. 11, but other equivalent circuits can give greater inaccuracies.

Acknowledgment

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